Mavzu: Aniq integralning tatbiqlari

Reja:

1. Aniq integralning geomatriyaga tatbiqlari (yuza va hajmni hisoblash)
2. Aniq integralning fizika va amexanikaga tatbiqlari (kuchning bajargan ishi va ismning bosib o`tgan yo`lini hisoblash)

1. Aniq integralning geomatriyaga tatbiqlari

Yuzani dekart koordinatalarida hisoblash. Aniq integralning geometrik ma'nosiga asosan abssissalar o`qidan yuqorida yotgan, ya'ni yuqoridan $y = f(x)$ ($f(x) \geq 0$) funksiya grafigi bilan, quyidan $Ox$ o`q bilan, yon tomonlaridan $x = a$ va $x = b$ to`g`ri chiziqlar bilan chegaralangan egri chiziqli trapetsiyaning yuzasi

$$S = \int_a^b f(x)dx \qquad (1)$$

integtegralga teng bo`ladi[1].

Shu kabi, abssissalar o'qidan pastda yotgan, ya'ni quyidan $y = f(x)$ ($f(x) \leq 0$) funksiya grafigi bilan, yuqoridan Ox o`q bilan, yon tomonlaridan $x = a$ va $x = b$ to`g`ri chiziqlar bilan chegaralangan egri chiziqli trapetsiyaning yuzasi

$$S = -\int_a^b f(x)dx \qquad (2)$$

integtegralga teng bo`ladi.

(1) va (2) formulalarni bitta formula bilan umumlashtirish mumkin:

$$S = \left|\int_a^b f(x)dx\right|. \qquad (3)$$

Misol. $y = x^2$, $y = 0$ va $x = 1$ chiziqlar bilan chegaralangan tekis shakl (1-shakl) yuzasini (1) formula bilan topamiz:

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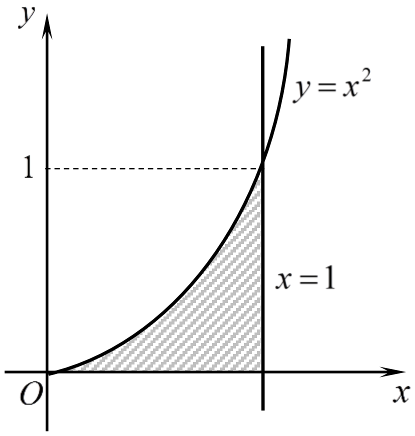
[1] **George B. Thomas, Ross L.Finney-Calculus and Analytic Geometry 1995 pp 393-400**

$$S=\int_0^1 x^2dx=\frac{x^3}{3}\Big|_0^1=\frac{1}{3}.$$

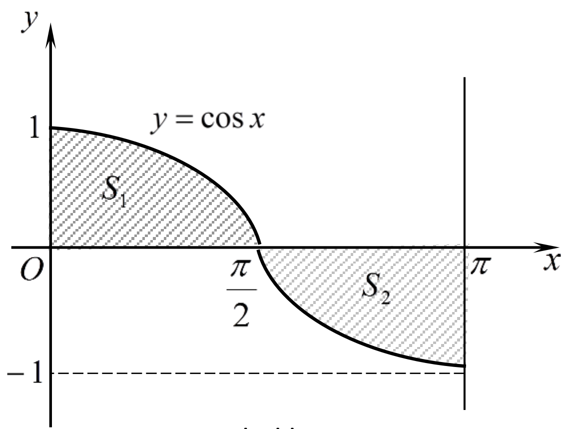
Yuzani hisoblashga oid murakkabroq masalalar yuzaning additivlik xossasiga asoslangan holda yechiladi. Bunda tekis shakl kesishmaydigan qismlarga ajratiladi va aniq integralning 4^o xossasiga ko`ra tekis shaklning yuzasi qismlar yuzalarining yig`indisiga teng bo`ladi.

Misol. $y=\cos x,\ y=0,\ x=0$ va $x=\pi$ chiziqlar bilan chegaralangan tekis shakl yuzasini hisoblaymiz. Bunda berilgan tekis shaklni yuzalari S_1 va S_2 bo`lgan kesishmaydigan qismlarga ajratamiz (2-shakl). U holda yuzaning additivlik xossasiga asosan berilgan tekis shaklning yuzasi qismlar yuzalarining yig`indisiga teng bo`ladi. Demak,

$$S=S_1+S_2=\int_0^{\frac{\pi}{2}}\cos xdx-\int_{\frac{\pi}{2}}^{\pi}\cos xdx=\sin x\Big|_0^{\frac{\pi}{2}}-\sin x\Big|_{\frac{\pi}{2}}^{\pi}=1-(-1)=2.$$



1- shakl.



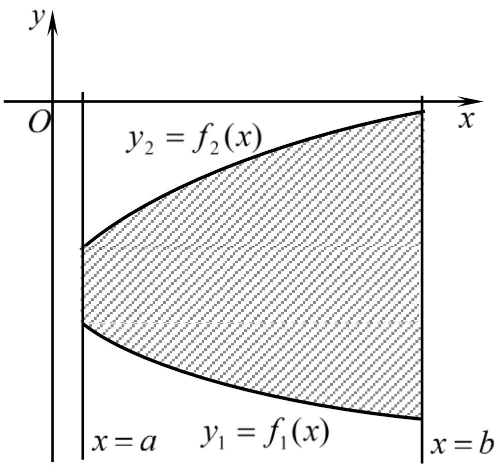
2-shakl.

$[a;b]$ kesmada ikkita $y_1=f_1(x)$ va $y_2=f_2(x)$ uzliksiz funksiyalar berilgan va $x\in[a;b]$ da $f_2(x)\geq f_1(x)$ bo`lsin. Bu funksiyalarning grafiklari va $x=a$, $x=b$ to`g`ri chiziqlar bilan chegaralangan tekis shaklning yuzasini topamiz.

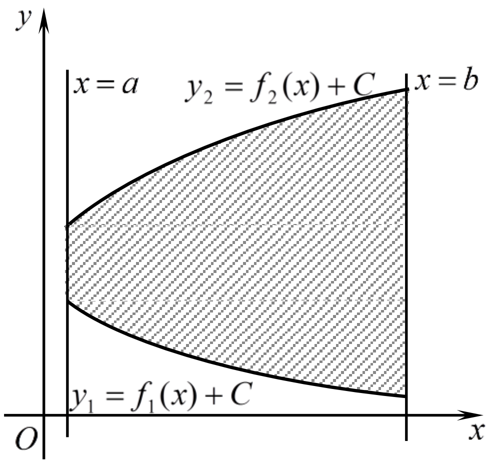
Har ikkala funksiya musbat bo`lganda bu tekis shaklning yuzasi yuqoridan $y_2=f_2(x)$ va $y_1=f_1(x)$ funksiyalar garfiklari bilan, quyidan $Ox$ o`q bilan, yon tomonlardan $x=a$ va $x=b$ to`g`ri chiziqlar bilan chegaralangan egri chiziqli trapetsiyalar yuzalarining ayirmasiga teng bo`ladi:

$$S = \int_a^b f_2(x)dx - \int_a^b f_1(x)dx = \int_a^b (f_2(x) - f_1(x))dx. \qquad (4)$$

(4) formula $[a;b]$ kesmada uzluksiz va musbat bo`lmagan $y_2 = f_2(x)$ va $y_1 = f_1(x)$ funksiyalar uchun ham o`rinli bo`ladi. Haqiqatan ham, agar $y_2 = f_2(x)$ va $y_1 = f_1(x)$ funksiyalar $[a;b]$ kesmada manfiy qiymatlar qabul qilsa (bunda $y_2 \geq y_1$) (3-shakl), har bir funksiyaga bir xil o`zgarmas $y = C$ qiymatlar qo`shish orqali $y_1 = f_1(x) + C$ va $y_2 = f_2(x) + C$ funksiyalar grafiklarini $Ox$ o`qidan yuqorida joylashtirish mumkin (4-



3-shakl



4-shakl

shakl).

4-shakldagi tekis shakl 3-shakldagi tekis shaklni parallel ko`chirish orqali hosil qilindi. Shu sababli yuzaning ko`chishga nisbatan invariantlik xossasiga ko`ra bu tekis shakllar teng yuzalarga ega bo`ladi. 4-shakldagi yuza uchun (4) formula o`rinli, ya’ni

$$S = \int_a^b (f_2(x) + C)dx - \int_a^b (f_1(x) + C)dx = \int_a^b ((f_2(x) + C) - (f_1(x) + C))dx.$$

Bundan $$S = \int_a^b (f_2(x) - f_1(x))dx.$$

Demak, (4) formula 3-shakldagi tekis shakl uchun ham o`rinli bo`ladi.

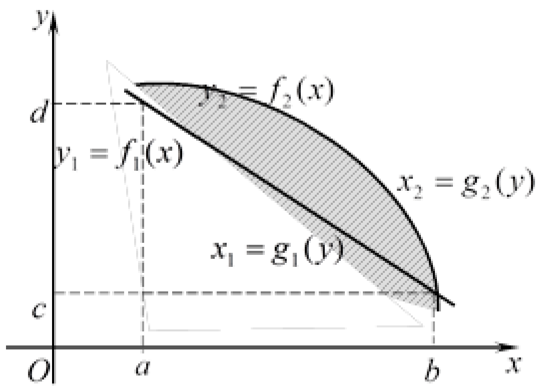
Ayrim hollarda yuzani hisoblashga oid masalalar yuzaning ko`chishga nisbatan invariantlik xossasidan foydalangan holda soddalashtiriladi. Bunda tekis shakl yuzasi (4) formulada x va y

o`zgaruvchilar ($Ox$ va $Oy$ o`qlar) ning o`rnini almashtirish yo`li bilan hisoblanadi (5-shakl), ya’ni

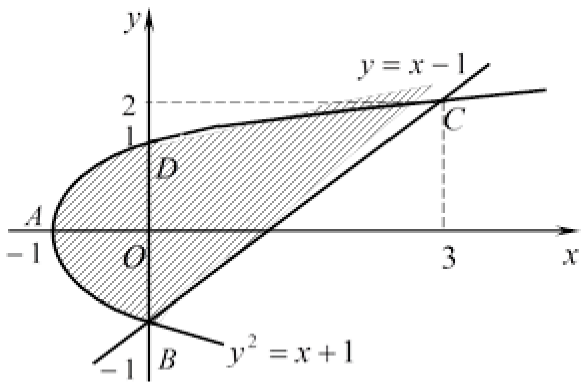
$$S = \int_a^b (f_2(x) - f_1(x))dx = \int_c^d (g_2(y) - g_1(y))dy. \qquad (5)$$

Misollar

1. $y^2 = x+1$ va $y = x-1$ chiziqlar bilan chegaralangan tekis shaklning yuzasini hisoblaymiz. Tekis shakl umumiy $B(0;-1)$ va $C(3;2)$ nuqtalarga ega bo`lgan parabola va to`g`ri chiziq bilan chegaralangan. Tekis shaklni uchta qismga, ya’ni yuzalari $S_1$ ga teng bo`lgan AOD va AOB parabolik sektorlarga va yuzasi S_2 ga teng bo`lgan BCD parabolik uchburchakka ajratamiz (6-shakl).



5-shakl



6-shakl

Bunda (1) va (4) formulalarni qo`llab, topamiz:

$$S = 2S_1 + S_2 = 2\int_{-1}^{0} \sqrt{x+1}dx + \int_0^3 (\sqrt{x+1} - (x-1))dx =$$

$$= \frac{4}{3}\sqrt{(x+1)^3}\Big|_{-1}^{0} + \left(\frac{2}{3}\sqrt{(x+1)^3} - \frac{x^2}{2} + x\right)\Big|_0^3 = \frac{9}{2}.$$

Bu yuzani y o`zgaruvchi bo`yicha hisoblanganda tekis shaklni qismlarga ajratiish shart bo`lmaydi.

2. $x = \frac{1}{2}y^2$, $y = -3$, $y = 1$ chiziqlar va ordinatalar o`qi bilan chegaralangan tekis shakl yuzasini hisoblaymiz:

$$S = \int_{-3}^{1} \frac{1}{2}y^2 dy = \frac{1}{2}\cdot\frac{1}{3}y^3\Big|_{-3}^{1} = \frac{1}{6}(1+27) = \frac{14}{3}.$$

Agar egri chiziqli trapetsiya yuqoridan $x=\varphi(t),\quad y=\psi(t),\quad \alpha\le t\le\beta$ parametrik tenglamalar bilan berilgan funksiya grafigi bilan chegaralangan bo`lsa (1) formulada $x=\varphi(t),\ dx=\varphi'(t)dt$ o`rniga qo`yish orqali o`zgaruvchi almashtiriladi.

U holda

$$S=\int_{\alpha}^{\beta}\psi(t)\varphi'(t)dt \qquad (6)$$

bo`ladi, bu yerda, $a=\varphi(\alpha)$ va $b=\varphi(\beta)$.

Misol. Radiusi R ga teng doira yuzasini hisoblaymiz. Buning uchun koordinatalar boshini doiraning markaziga joylashtiramiz. Bu doiraning aylanasi $x=R\cos t,\ \ y=R\sin t$ parametrik tenglamalar bilan aniqlanadi va doira koordinata o`qlariga nisbatan simmetrik bo`ladi. Shu sababli uning birinchi chorakdagi yuzasini hisoblaymiz (bunda x o`zgaruvchi $0$ dan $R$ gacha o`zgarganda t parametr $\frac{\pi}{2}$ dan 0 gacha o`zgaradi) va natijani to`rtga ko`paytiramiz:

$$S=4S_1=4\int_{\frac{\pi}{2}}^{0}R\sin t(-R\sin t)dt=4R^2\int_{0}^{\frac{\pi}{2}}\sin^2 tdt=$$

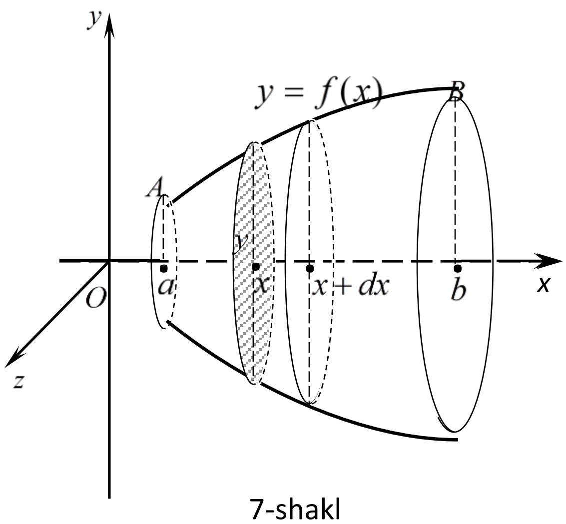
$$=2R^2\int_{0}^{\frac{\pi}{2}}(1-\cos 2t)dt=2R^2\left(t-\frac{\sin 2t}{2}\right)\Bigg|_{0}^{\frac{\pi}{2}}=\pi R^2.$$

Aylanish sirti yuzasini hosoblash.

AB egri chiziq $y=f(x)\ge 0$ funksiyaning grafigi bo`lsin. Bunda $x\in[a;b],\ \ y=f(x)$ funksiya va uning $y'=f'(x)$ hosilasi bu kesmada uluksiz bo`lsin.

AB egri chiziqning Ox o`q atrofida aylanishidan hosil bo`lgan jism sirti yuzasini hisoblaymiz. Buning uchun II sxemani qo`llaymiz.

1^o. Istalgan $x \in [a;b]$ nuqta orqali Ox o`qqa perpendikulyar tekislik o`tkazamiz. Bu tekislik aylanish sirtini radiusi $y = f(x)$ bo`lgan aylana bo`ylab kesadi (7-shakl). Bunda aylanish sirtidan iborat S kattalik x ning funksiyasi bo`ladi: $S = S(x)$ $(S(a) = 0$ va $S(b) = S)$.



7-shakl

2^o. x argumentga $\Delta x = dx$ orttirma beramiz va $x + \Delta x \in [a;b]$ nuqta orqali Ox o`qqa perpendikulyar tekislik o`tkazamiz. Bunda $S = S(x)$ funksiya «belbog`» ko`rinishida ΔS orttirma oladi.

Kesimlar orasidagi jismni yasovchisi dl bo`lgan va asoslarining radiuslari $y$ va $y + dy$ bo`lgan kesik konus bilan almashtiramiz. Bu kesik konusning yon sirti $dS = \pi(y + y + dy)dl = 2\pi ydl + \pi dydl$ ga teng. $dydl$ ko`paytmani dS ga nisbatan yuqori tartibli cheksiz kichik sifatida tashlab yuboramiz: $dS = 2\pi ydl$. Bunda $dl = \sqrt{1 + (y'_x)^2}dx$ ekanini hisobga olamiz: $dS = 2\pi y\sqrt{1 + (y'_x)^2}dx.$

3^o. dS ni a dan b gacha integrallab, topamiz:

$$S = 2\pi\int_a^b y\sqrt{1 + (y'_x)^2}dx \qquad (7)$$

Shu kabi $x = g(y)$, $y \in [c;d]$ funksiya grafigining Oy o`q atrofida aylantirshdan hosil bo`lgan jism sirtining yuzasi ushbu

$$S = 2\pi\int_c^d x\sqrt{1 + (x'_y)^2}dy \qquad (8)$$

formula bilan hisoblanadi.

Agar sirt $x = \varphi(t)$, $y = \psi(t)$, $\alpha \le t \le \beta$ parametrik tenglamalar bilan berilgan bo`lsa, u holda $AB$ egri chiziqning $Ox(Oy)$ o`q atrofida aylanishidan hosil bo`lgan jism sirti yuzasi quyidagicha hisoblanadi:

$$S = 2\pi\int_\alpha^\beta \psi(t)\sqrt{\varphi'^2(t) + \psi'^2(t)}dt \quad \left(S = 2\pi\int_{\alpha_1}^{\beta_1} \varphi(t)\sqrt{\psi'^2(t) + \varphi'^2(t)}dt\right), \quad .. \quad (9)$$

bu yerda $a = \varphi(\alpha)$ va $b = \varphi(\beta)$ $(c = \psi(\alpha_1)$va $d = \psi(\beta_1))$.

AB egri chiziq qutb koordinatalar sistemasida $r = r(\varphi)$, $\alpha \le \varphi \le \beta$ tenglama bilan berilgan bo`lganida quyidagi formulalar o`rinli bo`ladi:

$$S = 2\pi\int_{\alpha}^{\beta} r\sin\varphi\sqrt{r^2 + r'^2}\,d\varphi \ (Ox),\ S = 2\pi\int_{\alpha}^{\beta} r\cos\varphi)\sqrt{r^2 + r'^2}\,d\varphi \ (Oy) \quad .10)$$

Misollar. **1**. Radiusi R ga teng bo`lgan shar sirti yuzaini hisoblaymiz. Shar parametrik tenglamasi $x = R\cos t$, $y = R\sin t$ bo`lgan yarim aylananing Ox o`q atrofida aylanishidan hosil bo`ladi. Sharning koordinata o`qlariga simmetrik bo`lishini inobatga olib hisoblaymiz:

$$S = 2\cdot 2\pi\int_0^{\frac{\pi}{2}} R\sin t\sqrt{(-R\sin t)^2 + (R\cos t)^2}\,dt =$$

$$= 4\pi R^2\int_0^{\frac{\pi}{2}}\sin t dt = -4\pi R^2\cos t\Big|_0^{\frac{\pi}{2}} = 4\pi R^2 .$$

2. $y = ach\dfrac{x}{2}$ zanjir chizig`i $x = 0$ dan $x = a$ gacha bo`lagining Ox o`qi atrofida aylanishidan hosil bo`lgan sirt yuzasini hisoblaymiz (8-shakl).

Buning uchun avval $y' = sh\dfrac{x}{a}$ hosilani va $\sqrt{1+(y'_x)^2} = ch\dfrac{x}{a}$ ifodani topamiz.

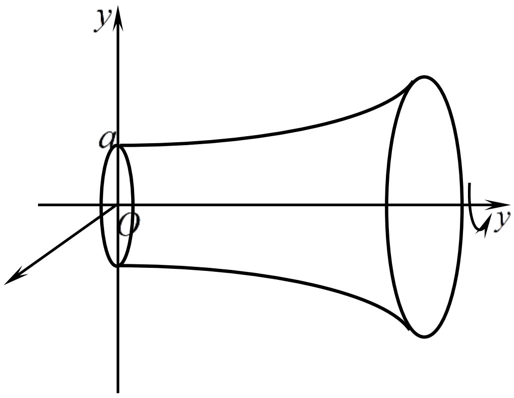
U holda (7) formulaga ko`ra

$$S = 2\pi\int_0^a ach^2\frac{x}{a}dx = \pi a\int_0^a\left(1 + ch\frac{2x}{a}\right)dx =$$

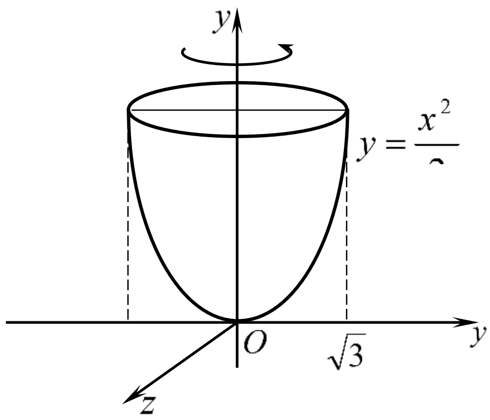
$$= \pi a\left(\frac{a}{2}sh\frac{2x}{a} + x\right)\Bigg|_0^a = \pi a^2\left(\frac{1}{2}sh2 + 1\right).$$

3. $y = \dfrac{x^2}{2}, x > 0$ parabola bo`lagining $y = \dfrac{3}{2}$ to`g`ri chiziq bilan kesilgan qismining $Oy$ o`qi atrofida aylanishidan hosil bo`lgan sirt yuzasini hisoblaymiz (9-shakl). Misol shartidan topamiz: $x = \sqrt{2y}$, $x' = \dfrac{1}{\sqrt{2y}}$. (8) formula bilan topamiz:

$$\sigma = 2\pi\int_0^{\frac{3}{2}}\sqrt{2y}\sqrt{1+\frac{1}{2y}}dy = 2\pi\int_0^{\frac{3}{2}}\sqrt{2y+1}dy = 2\pi\frac{1}{3}\frac{1}{\frac{3}{2}}(2y+1)^{\frac{3}{2}}\Big|_0^{\frac{3}{2}} = \frac{14\pi}{3}.$$



8-shakl



9-shakl

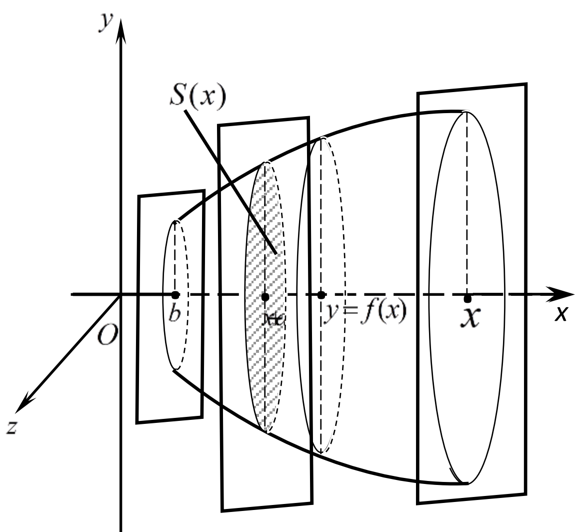
2.

Hajmlarni hisoblash

Hajmni ko`ndalang kesim yuzasi bo`yicha hisoblash

Hajmi hisoblanishi lozim bo`lgan qandaydir jism (10-shakl) uchun uning istalgan ko`ndalang kesim yuzasi S ma'lum bo`lsin. Bu yuza ko`ndalang kesim joylashishiga bog`liq bo`ladi: $S = S(x)$, $x \in [a;b]$, bu yerda $S(x)$- $[a;b]$ kesmada uzluksiz funksiya. Izlanayotgan hajmni topamiz.

1^o. Istalgan $x \in [a;b]$ nuqta orqali Ox o`qqa perpendikulyar tekislik o`tkazamiz. Jismning bu tekislik bilan kesimi yuzasini $S(x)$ bilan va jismning bu tekislikdan chapda yotgan bo`lagining hajmini $V(x)$ bilan belgilaymiz (10-shakl). Bunda $V$ kattalik $x$ ning funksiyasi bo`ladi: $V = V(x)$ $(V(a) = 0$ va $V(b) = V)$.



10-shakl

2^o. $V(x)$ funksiyaning dV differensialini topamiz. Bu differensial Ox o`q bilan $x$ va $x + \Delta x$ nuqtalarda kesishuvchi parallel tekisliklar orasidagi «elementar qatlam» dan iborat bo`ladi. Bu differensialni asosi $S(x)$ ga va balandligi dx ga teng silindr bilan taqriban almashtirish mumkin. Demak, $dV = S(x)dx.$

3^o. dV ni a dan b gacha integrallab, izlanayotgan hajmni topamiz:

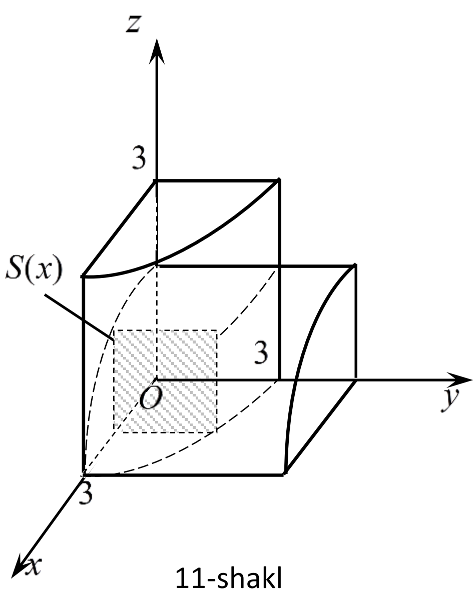
$$V = \int_a^b S(x)dx. \tag{11}$$

Misollar. 1. $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ ellipsoidning hajmini hisoblaymiz.

Ellipsoidning koordinatalar boshidan $x\ (-a \le x \le a)$ masofada o`tuvchi $Ox$ o`qqa perpendikulyar tekislik bilan kesamiz. Kesimda yarim o`qlari $b(x) = b\sqrt{1 - \frac{x^2}{a^2}}$ va $c(x) = c\sqrt{1 - \frac{x^2}{a^2}}$ bo`lgan ellips hosil bo`ladi. Uning yuzasi $S(x) = \pi b(x)c(x) = \pi bc\left(1 - \frac{x^2}{a^2}\right)$. U holda

$$V = \int_{-a}^{a} \pi bc\left(1 - \frac{x^2}{a^2}\right)dx = \pi bc\left(x - \frac{x^3}{3a^2}\right)\Bigg|_{-a}^{a} = \frac{4}{3}\pi abc.$$

2. $x^2 + y^2 = 9$ va $x^2 + z^2 = 9$ silindrlar bilan chegaralangan jism hajmini hisoblaymiz. 11-shakda berilgan jismning I oktantda $(x \ge 0, y \ge 0, z \ge 0)$ joylashgan sakkizdan bir bo`lagi keltirilgan. Uning $Ox$ o`qqa perpendikulyar tekislik bilan kesimi kvadratdan iborat. Kesim abssissasi $(x;0;0)$ nuqtadan o`tganda kvadratning tomonlari $a = y = z = \sqrt{9 - x^2}$ ga va yuzasi $S(x) = 9 - x^2$ teng bo`ladi, bu yerda $0 \le x \le 9.$



11-shakl

Jismning hajmni (11) formula bilan hisoblaymiz:

$$V = 8\int_0^3 (9 - x^2)dx = 8\left(9x - \frac{x^3}{3}\right)\Bigg|_0^3 = 144.$$

Aylanish jismlarining hajmini hisoblash

Yuqoridan $y = f(x)$ uzluksiz funksiya grafigi bilan, quyidan Ox o`q bilan, yon tomonlaridan $x = a$ va $x = b$ to`g`ri chiziqlar bilan chegaralangan egri chiziqli trapetsiyaning $Ox$ o`q atrofida aylantirishdan hosil bo`lgan jism hajmini hisoblaymiz. Bu jismning ixtiyoriy ko`ndalang kesimi doiradan iborat. Shu sababli jismning $X = x$ tekislik bilan kesimining yuzasi $S(x) = \pi y^2$ bo`ladi.

U holda (11) formulaga ko`ra

$$V = \pi\int_a^b y^2 dx. \tag{12}$$

Shu kabi yuqoridan $y = f(x)$ uzluksiz funksiya grafigi bilan, quyidan Ox o`q bilan, yon tomonlaridan $x = a$ va $x = b$ to`g`ri chiziqlar bilan chegaralangan egri chiziqli trapetsiyani $Oy$ o‘qi atrofida aylantirishdan hosil bo`lgan jismning hajmi quyidagi formula bilan hisoblanadi:

$$V = 2\pi\int_a^b yxdx. \tag{13}$$

Agar egri chiziqli trapetsiya $x = \varphi(y)$ uzluksiz funksiya grafigi, Oy o`qi,

$y = c$ va $y = d$ to`g`ri chiziqlar bilan chegaralangan bo`lsa, u holda

$$V = \pi\int_c^d x^2 dy \ (Oy) \quad \left(V = 2\pi\int_c^d xydy \ (Ox)\right) \tag{14}$$

bo`ladi.

$r = r(\varphi)$ egri chiziq va $\varphi = \alpha$, $\varphi = \beta$ nurlar bilan chegaralangan egri chiziqli sektorning qutb o`qi atrofida aylanishidan hosil bo`lgan jismning hajmi

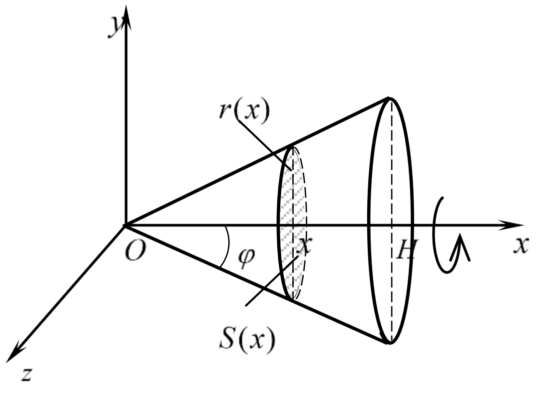
$$V = \frac{2\pi}{3}\int_\alpha^\beta r^3 \sin\varphi d\varphi \tag{15}$$

formula bilan topiladi.

Misollar. **1.** $x = y^2, y = 0$ va $x = a \ (a > 0)$ chiziqlar bilan chegaralangan tekis shaklning Ox o`q atrofida aylanishidan hosil bo`lgan jismning hajmini (12) formula bilan hisoblaymiz:

$$V = \pi\int_0^a (\sqrt{x})^2 dx = \pi\int_0^a xdx = \pi\frac{x^2}{2}\Bigg|_0^a = \frac{\pi a^2}{2}.$$

2. Radiusi R ga va balandligi H ga teng bo`lgan konusning hajmini hisoblaymiz. Bunda konusnni katetlari $R$ va $H$ bo`lgan to`g`ri burchakli uchburchakning balandlik bo`ylab



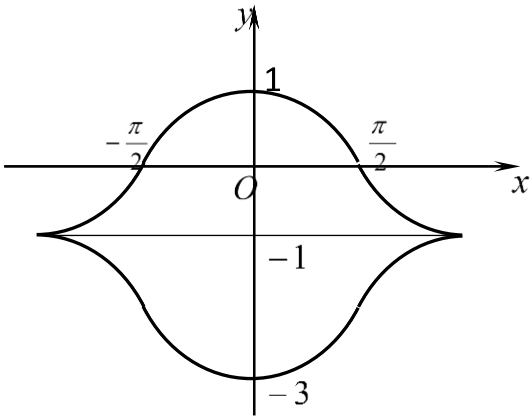
20-shakl

yo`nalgan $Ox$ o`q atrofida aylanishidan hosil bo`lgan jism deyish mumkin (20-shakl). Gipotenuza tenglamasi $y = kx$ bo`lsin, holda

$$y = kx\,,\quad k = tg\varphi = \frac{R}{H},\quad y = \frac{R}{H}x$$

Bundan $V = \pi\int_0^H y^2dx = \pi\int_0^H \frac{R^2}{H^2}x^2dx = \frac{\pi R^2}{H^2}\cdot\left.\frac{x^3}{3}\right|_0^H = \frac{1}{3}\pi R^3 H.$

3. $y = \cos x$ va $y = -1$ chiziqlar bilan chegaralangan tekis shaklning $-\pi \le x \le \pi$ da $y = -1$ to`g`ri chiziq atrofida aylanishidan hosil bo`lgan jismning hajmini hisoblaymiz. Berilgan chiziqlar bilan chegaralangan tekis shaklning aylanishidan hosil bo`lgan jism 21-shaklda keltirilgan. Egri chiiq $y = -1$ to`g`ri chiziq atrofida aylangani uchun yangi koordinatalar sistemasiga o`tish maqsadga muvofiq bo`ladi: $x' = x,\ y' = y + 1$.U holda aylanish jismining hajmi



21-shakl

$$V = \pi\int_{-\pi}^{\pi}(y')^2dx' = \pi\int_{-\pi}^{\pi}(y+1)^2dx =$$

$$= \pi\int_{-\pi}^{\pi}(\cos x + 1)^2dx = \ = \pi\int_{-\pi}^{\pi}(1 + 2\cos\varphi + \cos^2\varphi)d\varphi =$$

$$= \pi(\varphi + 2\sin\varphi)\Big|_{-\pi}^{\pi} + \frac{1}{2}\pi\int_0^{\pi}(1 + \cos 2\varphi)d\varphi = 2\pi^2 + \frac{1}{2}\pi\left(\varphi + \frac{1}{2}\sin 2\varphi\right)\Bigg|_{-\pi}^{\pi} = 3\pi^2.$$

2. Aniq integralning fizika va amexanikaga tatbiqlari.

Kuchning bajargan ishini hisoblash

Moddiy nuqta o`zgaruvchan $\vec{F}$ kuch ta'sirida $Ox$ o`qi bo`ylab harakatlanayotgan bo`lsin va bunda kuchning yo`nalishi harakat yo`nalishi bilan bir xil bo`lsin. U holda $\vec{F}$ kuchning moddiy nuqtani $Ox$ o`qi bo`ylab $x=a$ nuqtadan $x=b$ $(a<b)$ nuqtaga ko`chirishda bajargan ishi quyidagi formula bilan hisoblanadi:

$$A=\int_a^b F(x)dx \qquad (16)$$

bu yerda $F(x)$ funksiya $[a;b]$ kesmada uzluksiz.

Misollar. 1. Agar prujina 1H kuch ostida $1\ sm$ ga cho`zilsa, uni $6\ sm$ cho`zish uchun qancha ish bajarish kerak bo`lishini topamiz. Guk qonuniga muvofiq $F$ kuch va $x$ cho`zilish o`zaro $F=kx$ bog`lanishga ega. Proporsionallik koeffitsiyentini masalaning shartidan topamiz:

$x=1\ sm=0{,}01\ m$ da $F=1H$, ya'ni $1=k\cdot 0{,}01$.

Bundan, $k=100$ va. $F=100x$

U holda

$$A=\int_0^{0,06} 100x dx=50x^2\Big|_0^{0,06}=0{,}18\ (J)$$

2. m massali kosmik kemani erdan h masofaga uchurish uchun qancha ish bajarish kerak bo`lishini topamiz. Butun olam tortishish qonuniga ko`ra yerning jismni tortish kuchi $F=k\dfrac{mM}{x^2}$ ga teng, bu yerda M-yerning massasi, x- yer markazidan kosmik kemagacha bo`lgan masofa, k- gravtasiya doimiyligi. Yer sirtida, ya'ni $x=R$ da $F=mg$ ga teng, bu yerda g - erkin tushish tezlanishi. U holda

$$mg=k\frac{mM}{R^2}.$$

Bundan $kM=gR^2$ va $F=mg\dfrac{R^2}{x^2}$.

Izlanayotgan ishni (16) formula bilan topamiz:

$$A=\int_R^{R+h} mg\frac{R^2}{x^2}dx=-mgR^2\frac{1}{x}\Big|_R^{R+h}=-mgR^2\left(\frac{1}{R+h}-\frac{1}{R}\right)=mgR\frac{h}{R+h}.$$

Agar kosmik kema cheksizlikka ketsa, ya'ni $h\to\infty$ da $A=mgR$ bo`ladi.

3. Ikkita e_0 va e elektr zaryadi mos ravishda Ox o'qining $x_0=0$ va $x_1=a$ nuqtalrida joylangan. Ikkinchi zaryadni $x_2=b\ (b>a)$ masofaga ko`chirish uchun kerak bo`ladigan ishni topamiz. Kulon qonuniga ko`ra e_0 zaryad e zaryadni $F=\frac{e_0e}{x^2}$ kuch bilan itaradi, bu yerda x-zaryadlar orasidagi masofa.

Izlanayotgan ishni (16) formula bilan topamiz:

$$A=\int_a^b e_0e\frac{dx}{x^2}=-e_0e\frac{1}{x}\Big|_a^b=-e_0e\left(\frac{1}{b}-\frac{1}{a}\right)=\frac{e_0e(b-a)}{ab}.$$

Jismning bosib o`tgan yo`li

Moddiy nuqta (jism) to`g`ri chiziq bo`ylab o`zgaruvchan $v=v(t)$ tezlik bilan harakatlanayotgan bo`lsin. Bu nuqtaning $t_1$ dan $t_2$ gacha vaqt oralig`ida bosib o`tgan yo`lini topamiz.

Hosilaning fizik ma'nosiga ko`ra nuqtaning bir tomonga harakatida «to`g`ri chiziqli harakat tezligi yo`ldan vaqt bo`yicha olingan hosilaga teng», ya'ni $v(t)=\frac{dS}{dt}$. Bundan $dS=v(t)dt$. Bu tenglikni t_1 dan t_2 gacha integrallaymiz:

$$S=\int_{t_1}^{t_2} v(t)dt. \qquad (17)$$

Izoh. Bu formulani aniq integralni qo`llash sxemalari bilan topish mumkin.

Misol. Moddiy nuqtaning tezligi $v=2(6-t)\ m/s$ qonun bilan o`zgaradi. Nuqtaning harahat boshidan eng katta uzoqlashishini topamiz:

$$S=\int_0^t 2(6-t)dt=12t-t^2.$$

Nuqtaning eng katta uzoqlashishini yo`lni vaqtning funksiyasi sifatida qarab, topamiz: $S'=12-2t.$ $t=6$ da $S'=0$ bo`ladi. Bundan

$$S_{\max} = 12 \cdot 6 - 6^2 = 36 \ m.$$